

ESR Curriculum for the ITN - Deliverable D4.5

Prerequisites

- Quantum Field Theory including Feynman diagrams and traditional perturbative methods

Basics of Amplitudes

- Kinematics and building blocks (color decomposition, spinor helicity formalism, twistor variables, momentum twistors)
- Tree-level technology, in particular recursions (off-shell currents (Berends-Giele), on-shell (BCFW), CSW rules)

Loop-Level Technologies

- Amplitude decompositions (Master Integrals, IBP, Laporta algorithm)
- Computation of integral coefficients (unitarity, generalised unitarity, D-dimensional methods, OPP algorithm)
- Integration and summation strategies (Feynman and Mellin-Barnes representations, differential and difference equations, symbolic summation and integration)
- Mathematical structure of loop amplitudes (generalized polylogarithms, symbols, Hopf algebra structure, iterative and non-iterative sums)
- Computer algebra tools (FORM, Mathematica, FeynArts, FormCalc)

Specialised topics in Amplitudes

- Factorisation properties of amplitudes (soft/collinear)
- IR cancellation between real and virtual contributions
- Landau singularities and Landau equations
- Regge Kinematics
- Collider physics (extracting predictions from amplitude calculations)
- Twistor actions
- Basics of Grassmannian geometry
- Near-collinear kinematics and the Pentagon Operator Product Expansion

Symmetries

- Super(conformal)symmetry (on shell superspace/superamplitudes, SUSY Ward identities, SUSY BCFW recursion)
- Hidden symmetries and integrability (Dual conformal symmetry, Yangians, 2d S-matrices, N=4 SYM, ABJM)
- Asymptotic symmetries and soft theorems

String Theory and String-inspired methods

- Basics of string amplitude computation (open and closed strings)
- KLT in string theory and field theory
- Color-kinematics duality and double copy
- CHY formulae
- AdS/CFT correspondence

Literature Guide

Books

Perturbative QCD:

- R. Keith Ellis, W. James Stirling, and B. R. Webber. QCD and collider physics. *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.*, 8:1–435, 1996
- John Collins. Foundations of perturbative QCD. *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.*, 32:1–624, 2011

On-shell methods:

- Henriette Elvang and Yu-tin Huang. Scattering amplitudes. *arXiv:1308.1697*, 2013
- Johannes M. Henn and Jan C. Plefka. Scattering Amplitudes in Gauge Theories. *Lect. Notes Phys.*, 883:pp.1–195, 2014

String theory:

- Michael B. Green, J. H. Schwarz, and Edward Witten. *Superstring Theory. Vol. 1: Introduction*. Cambridge Monographs on Mathematical Physics. 1988
- Michael B. Green, J. H. Schwarz, and Edward Witten. *Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies and Phenomenology*. 1988
- J. Polchinski. *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007
- J. Polchinski. *String theory. Vol. 2: Superstring theory and beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007
- Volker Schomerus. *A Primer on String Theory*. Cambridge University Press, 2017

Reviews/Pedagogical Lectures

General introduction to modern methods for amplitudes:

- Michelangelo L. Mangano and Stephen J. Parke. Multiparton amplitudes in gauge theories. *Phys. Rept.*, 200:301–367, 1991
- Lance J. Dixon. Calculating scattering amplitudes efficiently. In *QCD and beyond. Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics, TASI-95, Boulder, USA, June 4-30, 1995*, pages 539–584, 1996
- Lance J. Dixon. Scattering amplitudes: the most perfect microscopic structures in the universe. *J. Phys.*, A44:454001, 2011

Loop methods:

- Zvi Bern, Lance J. Dixon, and David A. Kosower. On-Shell Methods in Perturbative QCD. *Annals Phys.*, 322:1587–1634, 2007

- Ruth Britto. Loop Amplitudes in Gauge Theories: Modern Analytic Approaches. *J. Phys.*, A44:454006, 2011
- R. Keith Ellis, Zoltan Kunszt, Kirill Melnikov, and Giulia Zanderighi. One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts. *Phys. Rept.*, 518:141–250, 2012
- Johannes Blümlein and Carsten Schneider. Analytic computing methods for precision calculations in quantum field theory. *Int. J. Mod. Phys.*, A33(17):1830015, 2018

Generalised unitarity:

- Zvi Bern, Lance J. Dixon, and David A. Kosower. Progress in one loop QCD computations. *Ann. Rev. Nucl. Part. Sci.*, 46:109–148, 1996
- Harald Ita. Susy Theories and QCD: Numerical Approaches. *J. Phys.*, A44:454005, 2011

Differential equations:

- Johannes M. Henn. Lectures on differential equations for Feynman integrals. *J. Phys.*, A48:153001, 2015

Multuple Polylogs, Symbols, Hopf Algebra Structure:

- Claude Duhr. Mathematical aspects of scattering amplitudes. In *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (TASI 2014): Boulder, Colorado, June 2-27, 2014*, pages 419–476, 2015
- Erik Panzer. *Feynman integrals and hyperlogarithms*. PhD thesis, Humboldt U., Berlin, Inst. Math., 2015
- Lectures by Christian Vergu:
<http://www.maths.dur.ac.uk/lms/098/talks/0349vergu.pdf>

Twistor Methods:

- Freddy Cachazo and Peter Svrcek. Lectures on twistor strings and perturbative Yang-Mills theory. *PoS*, RTN2005:004, 2005
- Martin Wolf. A First Course on Twistors, Integrability and Gluon Scattering Amplitudes. *J. Phys.*, A43:393001, 2010
- Tim Adamo, Mathew Bullimore, Lionel Mason, and David Skinner. Scattering Amplitudes and Wilson Loops in Twistor Space. *J. Phys.*, A44:454008, 2011
- Tim Adamo. *Twistor actions for gauge theory and gravity*. PhD thesis, Cambridge U., DAMTP, 2013

Grassmannian Geometry: (Original Papers)

- Nima Arkani-Hamed, Jacob L. Bourjaily, Freddy Cachazo, Alexander B. Goncharov, Alexander Postnikov, and Jaroslav Trnka. *Grassmannian Geometry of Scattering Amplitudes*. Cambridge University Press, 2016
- Nima Arkani-Hamed and Jaroslav Trnka. The Amplituhedron. *JHEP*, 10:030, 2014

Regge Kinematics:

- J. Bartels, L. N. Lipatov, and A. Prygarin. Integrable spin chains and scattering amplitudes. *J. Phys.*, A44:454013, 2011
- J. Bartels. AdS/CFT: Scattering amplitudes in the Regge limit: From weak to strong coupling. *Int. J. Mod. Phys. Conf. Ser.*, 04:26–34, 2011

Methods for Supersymmetric Amplitudes:

- Luis F. Alday and Radu Roiban. Scattering Amplitudes, Wilson Loops and the String/Gauge Theory Correspondence. *Phys. Rept.*, 468:153–211, 2008
- J. M. Drummond. Hidden Simplicity of Gauge Theory Amplitudes. *Class. Quant. Grav.*, 27:214001, 2010
- John Joseph M. Carrasco and Henrik Johansson. Generic multiloop methods and application to N=4 super-Yang-Mills. *J. Phys.*, A44:454004, 2011

Susy-Ward Identities:

- Henriette Elvang, Daniel Z. Freedman, and Michael Kiermaier. SUSY Ward identities, Superamplitudes, and Counterterms. *J. Phys.*, A44:454009, 2011

Dual Conformal Symmetry and Yangian Symmetry:

- J. M. Drummond. Tree-level amplitudes and dual superconformal symmetry. *J. Phys.*, A44:454010, 2011

Asymptotic Symmetries and Soft theorems:

- Andrew Strominger. Lectures on the Infrared Structure of Gravity and Gauge Theory. 2017

Integrable theories:

- L. D. Faddeev. How algebraic Bethe ansatz works for integrable model. In *Relativistic gravitation and gravitational radiation. Proceedings, School of Physics, Les Houches, France, September 26-October 6, 1995*, pages pp. 149–219, 1996
- P. Dorey. Exact S matrices. In *Conformal field theories and integrable models. Proceedings, Eotvos Graduate Course, Budapest, Hungary, August 13-18, 1996*, pages 85–125, 1996
- Diego Bombardelli. S-matrices and integrability. *J. Phys.*, A49(32):323003, 2016

Integrability in AdS/CFT:

- Niklas Beisert et al. Review of AdS/CFT Integrability: An Overview. *Lett. Math. Phys.*, 99:3–32, 2012
- Diego et al. Bombardelli. An integrability primer for the gauge-gravity correspondence: An introduction. *J. Phys.*, A49(32):320301, 2016

Wilson loop/Pentagon Operator Product Expansion:

- Luis F. Alday, Davide Gaiotto, Juan Maldacena, Amit Sever, and Pedro Vieira. An Operator Product Expansion for Polygonal null Wilson Loops. *JHEP*, 04:088, 2011
- Benjamin Basso, Amit Sever, and Pedro Vieira. Space-time S-matrix and Flux tube S-matrix II. Extracting and Matching Data. *JHEP*, 01:008, 2014

Color-kinematics duality:

- Z. Bern, Lecture Notes, (Amplitudes 2018 Summer School)
- John Joseph M. Carrasco. Gauge and Gravity Amplitude Relations. In *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (TASI 2014): Boulder, Colorado, June 2-27, 2014*, pages 477–557. WSP, WSP, 2015

CHY: (Original Papers)

- Freddy Cachazo, Song He, and Ellis Ye Yuan. Scattering of Massless Particles: Scalars, Gluons and Gravitons. *JHEP*, 07:033, 2014
- Freddy Cachazo, Song He, and Ellis Ye Yuan. Scattering of Massless Particles in Arbitrary Dimensions. *Phys. Rev. Lett.*, 113(17):171601, 2014

String theory:

- Lectures by David Tong:
<http://www.damtp.cam.ac.uk/user/tong/string/string.pdf>

AdS/CFT:

- Ofer Aharony, Steven S. Gubser, Juan Martin Maldacena, Hiroshi Ooguri, and Yaron Oz. Large N field theories, string theory and gravity. *Phys. Rept.*, 323:183–386, 2000
- Eric D’Hoker and Daniel Z. Freedman. Supersymmetric gauge theories and the AdS / CFT correspondence. In *Strings, Branes and Extra Dimensions: TASI 2001: Proceedings*, pages 3–158, 2002
- Joao Penedones. TASI lectures on AdS/CFT. In *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (TASI 2015): Boulder, CO, USA, June 1-26, 2015*, pages 75–136, 2017

Classic Papers

- V. Bargmann and Eugene P. Wigner. Group Theoretical Discussion of Relativistic Wave Equations. *Proc. Nat. Acad. Sci.*, 34:211, 1948
- R. E. Cutkosky. Singularities and discontinuities of Feynman amplitudes. *J. Math. Phys.*, 1:429–433, 1960
- Steven Weinberg. Photons and Gravitons in S-Matrix Theory: Derivation of Charge Conservation and Equality of Gravitational and Inertial Mass. *Phys. Rev.*, 135:B1049–B1056, 1964
- Steven Weinberg. Infrared photons and gravitons. *Phys. Rev.*, 140:B516–B524, 1965
- Gerard 't Hooft and M. J. G. Veltman. Regularization and Renormalization of Gauge Fields. *Nucl. Phys.*, B44:189–213, 1972
- David J. Gross and Frank Wilczek. Ultraviolet Behavior of Nonabelian Gauge Theories. *Phys. Rev. Lett.*, 30:1343–1346, 1973; H. David Politzer. Reliable Perturbative Results for Strong Interactions? *Phys. Rev. Lett.*, 30:1346–1349, 1973
- Gerard 't Hooft and M. J. G. Veltman. One loop divergencies in the theory of gravitation. *Ann. Inst. H. Poincaré Phys. Theor.*, A20:69–94, 1974
- J. Wess and B. Zumino. Supergauge Transformations in Four-Dimensions. *Nucl. Phys.*, B70:39–50, 1974
- Lars Brink, John H. Schwarz, and Joel Scherk. Supersymmetric Yang-Mills Theories. *Nucl. Phys.*, B121:77–92, 1977
- K. G. Chetyrkin and F. V. Tkachov. Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops. *Nucl. Phys.*, B192:159–204, 1981
- Alexander M. Polyakov. Quantum Geometry of Bosonic Strings. *Phys. Lett.*, B103:207–210, 1981
- Michael B. Green, John H. Schwarz, and Lars Brink. N=4 Yang-Mills and N=8 Supergravity as Limits of String Theories. *Nucl. Phys.*, B198:474–492, 1982
- R. Kleiss and W. James Stirling. Spinor Techniques for Calculating p anti-p \rightarrow W $^{+-}$ / Z0 + Jets. *Nucl. Phys.*, B262:235–262, 1985; Zhan Xu, Da-Hua Zhang, and Lee Chang. Helicity Amplitudes for Multiple Bremsstrahlung in Massless Nonabelian Gauge Theories. *Nucl. Phys.*, B291:392–428, 1987
- H. Kawai, D. C. Lewellen, and S. H. H. Tye. A Relation Between Tree Amplitudes of Closed and Open Strings. *Nucl. Phys.*, B269:1–23, 1986
- Stephen J. Parke and T. R. Taylor. An Amplitude for n Gluon Scattering. *Phys. Rev. Lett.*, 56:2459, 1986
- Frits A. Berends and W. T. Giele. Recursive Calculations for Processes with n Gluons. *Nucl. Phys.*, B306:759–808, 1988
- V. P. Nair. A Current Algebra for Some Gauge Theory Amplitudes. *Phys. Lett.*, B214:215–218, 1988

- Zvi Bern and David A. Kosower. The Computation of loop amplitudes in gauge theories. *Nucl. Phys.*, B379:451–561, 1992; Zvi Bern, Lance J. Dixon, David C. Dunbar, and David A. Kosower. One loop n-point gauge theory amplitudes, unitarity and collinear limits. *Nucl. Phys.*, B425:217–260, 1994
- Juan Martin Maldacena. The Large N limit of superconformal field theories and supergravity. *Int. J. Theor. Phys.*, 38:1113–1133, 1999. [Adv. Theor. Math. Phys.2,231(1998)]
- Z. Bern, Lance J. Dixon, D. C. Dunbar, M. Perelstein, and J. S. Rozowsky. On the relationship between Yang-Mills theory and gravity and its implication for ultraviolet divergences. *Nucl. Phys.*, B530:401–456, 1998
- Stefano Catani. The Singular behavior of QCD amplitudes at two loop order. *Phys. Lett.*, B427:161–171, 1998; George F. Sterman and Maria E. Tejeda-Yeomans. Multi-loop amplitudes and resummation. *Phys. Lett.*, B552:48–56, 2003
- Edward Witten. Perturbative gauge theory as a string theory in twistor space. *Commun. Math. Phys.*, 252:189–258, 2004
- Ruth Britto, Freddy Cachazo, and Bo Feng. New recursion relations for tree amplitudes of gluons. *Nucl. Phys.*, B715:499–522, 2005; Ruth Britto, Freddy Cachazo, Bo Feng, and Edward Witten. Direct proof of tree-level recursion relation in Yang-Mills theory. *Phys. Rev. Lett.*, 94:181602, 2005

Soft skills curriculum

- Scientific writing
- Time management
- Good scientific practice
- How to survive in research
- Entrepreneurship
- Presentation skills
- STEM outreach
- Career planning inside and outside academia
- Grant writing
- Job interview and defence training
- Project management